

COMPLEX LAPLACIAN BASED CASCADE FORMULATION FOR FORMATION CONTROL OF LARGE MULTI-AGENT NETWORK

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ABSTRACT. In this paper, we consider a problem of formation control of large vehicle network and propose a systematic way to establish robust and efficient interaction between agents, referred as cascade formulation. The proposed formulation divides the network into smaller clusters and meta-cluster ensuring 2-rooted communication graph. We use complex Laplacian approach to provide control law for proposed formulation. Further, we provide sufficient conditions on proposed approach to ensure globally stable formation. We also demonstrate that the proposed formulation gives flexibility to constrain the eigenvalue spectra of overall closed-loop dynamics to ensure desired convergence rate and control input intensity. The paper also illustrates the robustness of the proposed formulation to some uncertainties such as loss in communication links and actuator failure. The effectiveness of the proposed approach is illustrated by simulating it for an example of vehicle network with thirty agents.

1. INTRODUCTION

Distributed coordination control problem in the area of networked multi-agent systems has received considerable attention of many researchers in recent years. This is due to its broad application areas in various research fields such as the formation control of unmanned air vehicles(UAV) [OS06, MH01], the cooperative control of mobile vehicles [JL03, Mur07, YAB01], the design of distributed moving sensor networks [ÖFL04], and so forth. Designing a distributed control law for minimization of control efforts and convergence time with limited communication range and restricted processing power are the key challenges in formation control which make it an important area of research under coordination control.

Formation control [JL03], [YAB01], [FM04] problems have a long history in the field of decentralized control of multi-agent systems [OSFM07]. Plethora of research is available suggesting methods to address issues related to consensus problems like collision avoidance [CSMOS03], coordination between agents under switching networks [OSM04], [QYG14] and communication delay [Ren08, LXL11, LXZ14], convergence time optimization by improving algebraic connectivity of graph Laplacian [QGY14, ML14, DGJ06], robustness to link and node failure [ZC14], time varying formation control [BASCdW], [RB05] consensus of agent with input saturation [SCLL13], [LLRX13], etc. Furthermore, different forms of frameworks and agreement problems with various types of agent dynamics like formation stabilization using graph Laplacian [OSM02], [LBF04], leader follower architecture [LBY03, DF10, DOK01], formation control of heterogeneous nonlinear agent using passivity framework [Arc07], Lyapunov approach [ÖEH01], planar formation using complex Laplacian [LWHF14], hierarchical formation control [JDSK15], [KC12] has been reported in the literature.

Recently, Zhiyun Lin [LWHF14] has reported an approach to achieve rigid planar formation control using complex Laplacian which states the necessary and sufficient algebraic and geometrical conditions to achieve a globally stable formation. The corresponding consensus algorithm considers bidirectional 2-rooted graph

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topology [WHLY12] with two co-leaders providing flexibility in four degrees of freedom as translation, rotation and scaling.

The main contribution of this paper is to propose a novel method for formation control of large multi-agent systems. The proposed control methodology incorporates on complex Laplacian theory. Multi-agent systems are characterized by reduced convergence rate due to information percolation issues in the network which becomes more significant as the dimension of the system increases. The nonlinearities like saturation pose limitations on control input and may also result in instability of the multi-agent system in certain conditions. To cope with such difficulties, the proposed methodology, namely, cascade formulation channelizes information flow in a structured manner to reduce convergence time. The proposed formulation divides large network into small clusters with independent control laws designed using complex Laplacian, which act as local distributed controller. The orientation and formation of clusters are controlled by meta-cluster which commands the co-leaders of clusters. The stabilization matrix that stabilizes the complex Laplacian is designed by formulating constrained optimization problem using genetic algorithm to control the algebraic connectivity and maximum eigenvalue of closed-loop dynamics. In addition, the 2-rooted topology and local distributed control used for individual clusters helps to handle the communication and actuation failure in the network.

The outline for rest of the paper is given as follows. Section II describes the preliminaries of graph theory and complex Laplacian with its necessary and sufficient conditions. In Section III, we propose an approach called *cascade formulation* to shape the information flow in large multi-agent systems which divides a large network into decoupled stable clusters with local decentralized control law. Section IV summarizes simulation results and comparative analysis of proposed formulation. The conclusions and open problems are discussed in Section V.

2. PRELIMINARIES ON GRAPH THEORY AND COMPLEX LAPLACIAN

2.1. Notations. The symbols \mathbb{C} , \mathbb{N} , \mathbb{N}_k denote the set of complex, natural, and a natural number greater than k , respectively. We use $\mathbb{C}^{n \times m}$ to denote a vector space of complex valued matrices with n rows and m columns. ι denotes imaginary unit as $\iota^2 = -1$. $\mathbf{1}_n$ represents n -dimensional column vector of ones. We use $\text{diag}(d_1, d_2, \dots, d_n)$ as a diagonal matrix of order n with diagonal entries d_1, d_2, \dots, d_n . For $c \in \mathbb{C}$, $\text{Re}(c)$ and $\text{Im}(c)$ represents the real and imaginary part of a complex number c , respectively. Consider a matrix $A \in \mathbb{C}^{n \times m}$, then A^T represents transpose on matrix A . For a square matrix $F \in \mathbb{C}^{n \times n}$, $\text{eig}(F) = \lambda_1, \lambda_2, \dots, \lambda_n \in \mathbb{C}$ represents eigenvalues of F and the largest eigenvalue of F is given as $\lambda_{\max}(F) = \max\{\text{Re}(\lambda_1), \text{Re}(\lambda_2), \dots, \text{Re}(\lambda_n)\}$.

2.2. Graph theory. The interaction topology between agents of networked multi-agent system is represented using bidirectional graph $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ with n nodes $\mathcal{V} = \{1, 2, \dots, n\}$ and edges $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$. The bidirectional graph \mathcal{G} is considered as special case of directed graph because the weights assigned to edges (i, j) and (j, i) are not same. Thus the weight matrix of bidirectional graph is not symmetric as in case of undirected graph. Let the set of neighbors of agent i is given as $\mathcal{N}_i = \{j \in \mathcal{V} : (j, i) \in \mathcal{E}\}$.

Next, we introduce two definitions from [WHLY12] which are important to prove further results of this paper and to select proper interaction between nodes of graph \mathcal{G} .

Definition 2.1. For a bidirectional graph \mathcal{G} , a node v in \mathcal{V} is said to be 2-reachable from a non-singleton set \mathcal{U} of nodes, if it is possible to reach node v from any node in \mathcal{U} after eliminating any one node except node v .

Definition 2.2. A bidirectional graph \mathcal{G} is said to be 2-rooted, if there exists a subset of two nodes, from which every other node is 2-reachable. These two nodes are termed as roots of the graph \mathcal{G} .

Interested readers can refer to [LWHF14], for the graphical explanation of these definitions. The complex Laplacian L for bidirectional graph \mathcal{G} is given as,

$$(2.1) \quad L(i, j) = \begin{cases} -w_{ij}, & \text{if } i \neq j \text{ and } j \in \mathcal{N}_i, \\ 0, & \text{if } i \neq j \text{ and } j \notin \mathcal{N}_i, \\ \sum_{j \in \mathcal{N}_i} w_{ij}, & \text{if } i = j, \end{cases}$$

where $w_{ij} \in \mathbb{C}$ is the complex weight associated with edge (i, j) . The definition of complex Laplacian L also ensures that the row sum should be equal to zero (i.e. it has atleast one eigenvalue at origin with the corresponding eigenvector $\mathbf{1}_n$).

2.3. Planar formation using complex Laplacian. Consider a group of n agents in the plane with an objective to achieve desired formation using distributed control laws that are implementable with local information like relative distance with neighbors. In complex Laplacian approach, the formation configuration or shape of final formation is represented by assigning location $\xi_i \in \mathbb{C}$ in complex plane to i^{th} agent of the group. This complex formation vector $\xi = [\xi_1, \xi_2, \dots, \xi_n]^T \in \mathbb{C}^n$ is referred to as *formation basis*. The F_ξ is a function that acts on the formation basis ξ along four degree-of-freedom (translation, rotation, and scaling) to steer the formation as per the requirement.

$$F_\xi = c_1 \mathbf{1}_n + c_2 \xi, \quad c_1, c_2 \in \mathbb{C}.$$

Here, the bidirectional graph \mathcal{G} with n nodes is used as *sensing graph* in which an edge (i, j) represents measure of relative position between agent j and agent i as $(z_j - z_i)$, where $z_j, z_i \in \mathbb{C}$ denotes the positions of j^{th} and i^{th} agents, respectively.

Suppose each agent i is modelled as fully actuated point mass with single-integrator kinematics as

$$(2.2) \quad \dot{z}_i = u_i,$$

where $u_i \in \mathbb{C}$ is the velocity control input and the saturation limits are considered as $v_{min} \leq \text{Re}(u_i) \leq v_{max}$, $v_{min} \leq \text{Im}(u_i) \leq v_{max}$, v_{min} and v_{max} are the minimum and maximum velocities, respectively and those are mostly depend on the actuator saturation limit.

The local distributed control law to achieve stable formation is given as

$$(2.3) \quad u_i = d_i \sum_{j \in \mathcal{N}_i} w_{ij} (z_j - z_i), \quad i = 1, 2, \dots, n,$$

where $d_i \in \mathbb{C}$ is a design parameter which decides the performance and global stability of formation and w_{ij} is the complex weight on edge (i, j) represented in complex Laplacian L . The overall dynamics of n agents system with control law (2.3) is

$$(2.4) \quad \dot{z} = -DLz,$$

where $z = [z_1, z_2, \dots, z_n]^T \in \mathbb{C}^n$, L is the complex Laplacian of \mathcal{G} , and $D = \text{diag}(d_1, d_2, \dots, d_n)$ is stabilizing diagonal matrix. The diagonal matrix D transforms the eigenvalues of (2.4) to the left half of the complex plane. The necessary and sufficient conditions to a design complex Laplacian L and stabilizing diagonal matrix D are given below.

2.4. Necessary and sufficient conditions. The necessary and sufficient conditions for construction of complex Laplacian and stabilizing matrix are stated in the following Lemmas [LWHF14].

Lemma 2.3. *Let the formation basis $\xi \in \mathbb{C}^n$ satisfies $\xi_i \neq \xi_j, \forall i, j$. Every equilibrium state of (2.4) forms a globally stable geometric formation F_ξ if and only if there exists matrices $D, L \in \mathbb{C}^{n \times n}$ satisfying $\text{eig}(-DL) \leq 0$, $L\xi = 0$, and $\text{rank}(L) = n - 2$.*

Lemma 2.4. *For a bidirectional graph \mathcal{G} and $\xi \in \mathbb{C}^n$ satisfying $\xi_i \neq \xi_j, \forall i, j$. The algebraic conditions, $\text{rank}(L) = n - 2$. and $L\xi = 0$ satisfies for all $L \in \mathbb{C}^{n \times n}$ if and only if \mathcal{G} is 2-rooted.*

The Lemma 2.3 presents necessary algebraic condition to guarantee stationary formation and Lemma 2.4 gives graphical sufficiency condition to satisfy necessary condition mentioned in Lemma 2.3. The stability of the closed-loop system (2.4) depends on eigenvalues of DL . As complex Laplacian L have its eigenvalues in whole complex plane, it is not always stable. Thus, it is important to design a diagonal matrix D which stabilizes the closed-loop dynamics.

Lemma 2.5. *If the bidirectional graph \mathcal{G} is 2-rooted then there exists a stabilizing matrix D for the system $\dot{z} = -Lz$ such that $\dot{z} = -DLz$ is stable.*

For the proofs of these lemmas the reader can refer to [LWHF14].

2.5. Performance analysis of consensus algorithm. The eigenvalues of the stabilized complex Laplacian matrix exhibits some important information about performance of consensus algorithm such as stability, convergence rate, control efforts, etc. Some properties of eigenvalues of complex Laplacian are

1. Complex Laplacian L for 2-rooted graph topology has two eigenvalues at origin with corresponding eigenvectors $\mathbf{1}_n$ and formation basis ξ .
2. Unlike a real-valued Laplacian, complex Laplacian may have its eigenvalues in left half of complex plane.
3. All eigenvalues of L can be shifted to right half of complex plane by pre-multiplying it with real valued invertible diagonal matrix D without affecting properties mentioned in 1[Bal70].

Let the stabilized complex Laplacian DL has its eigenvalues at

$$\lambda_1 = \lambda_2 = 0, \\ 0 < \text{Re}(\lambda_3) \leq \text{Re}(\lambda_4) \leq \dots \leq \text{Re}(\lambda_n).$$

The smallest non-zero eigenvalue of stabilized complex Laplacian matrix λ_3 can be considered as an extension to the concept of algebraic connectivity [Fie73] of real-valued Laplacian to complex Laplacian with 2-rooted graph topology and is used as a measure of performance of collective dynamics. The largest eigenvalue of DL , λ_n , is used as a measure of the intensity of control signal [AF90]. It is very important to limit control signal magnitude (i.e. by limiting λ_n to certain value), as it can produce instability due to control inputs saturation.

Our main objective is to strengthen the algebraic connectivity of the complex Laplacian while limiting intensity of the control inputs to improve the speed of convergence in large multi-agent system. This is achieved by converting consensus problem into special formulation by cascading small clusters as discussed in the next section.

3. CASCADE FORMULATION

The complex Laplacian based consensus algorithm have an issue of increased convergence time as the system complexity increases. This pertains to the percolation of leader's information in the network. The section also discusses robustness of complex Laplacian. The robustness can be handled by implementing the proposed systematic formulation referred as *Cascade Formulation* in this section.

3.1. Limitations of complex Laplacian based consensus algorithm. The complex Laplacian based consensus algorithm requires 2-rooted graph topology with two roots acting as co-leaders for orientation and scaling of formation. In the large networked systems, the leader's information requires more time to percolate in to the network due to the increase in the number of agents, communication links and complexity of network. This affects the convergence time and the over all performance of the system. It is observed that as the number of agents increases, $\lambda_{max}(DL)$ increases which may generates the instability due to saturation of control inputs [HL01], [TGG07]. The system can be stabilized by scaling down the complex Laplacian by appropriate factor k as kDL , where $k \in (0, 1)$. However it affects the algebraic connectivity of network which in turn affects the

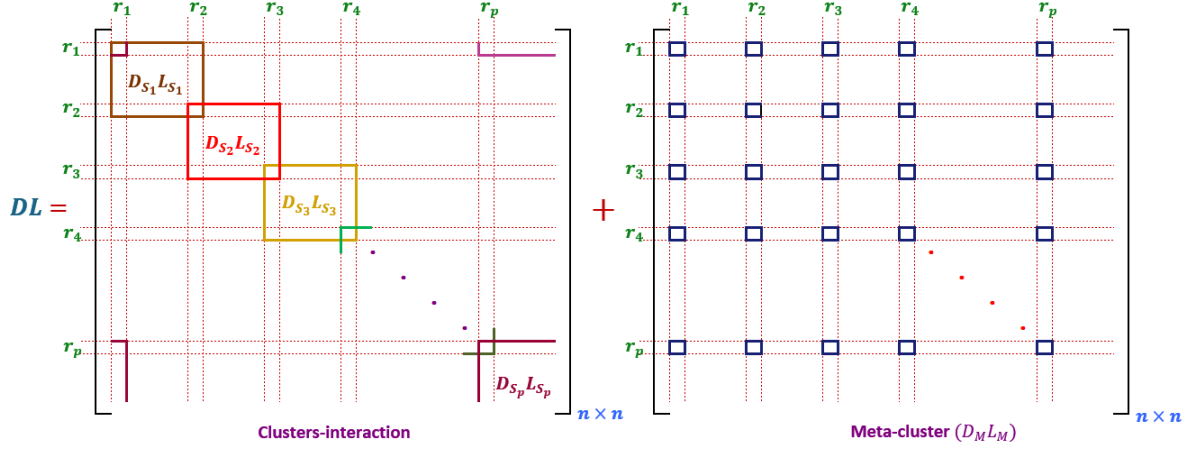


FIGURE 1. Representation of complex Laplacian structure using cascade formulation

convergence time. This sets the trade-off between convergence time and the control input intensity rendering the design of stabilizing matrix D , a tedious task.

Another major issue with complex Laplacian based control law is the robustness to communication and actuation failure of an agent. The overall dynamics leads to instability in case of failure of any communication link or actuator of an agent due to interruption in information flow.

3.2. Cascade Formulation. Consider finite but sufficiently large network of agents represented by bidirectional graph $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ with n nodes $\mathcal{V} = \{1, 2, \dots, n\}$ and edges $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$. In this paper, we propose a systematic approach, namely, *cascade formulation* to establish inter-agent interconnection to overcome aforementioned issues. The proposed formulation divides large multi-agent system with n agents into p small clusters denoted by $S_i^{q_i \times q_i}$, where $i = 1, 2, \dots, p$ and $q_i \in \mathbb{N}_3$ is a number of agents in each cluster. Each cluster $S_i^{q_i \times q_i}$ satisfies the properties of 2-rooted graph topology (i.e every cluster has $q_i - 2$ follower agents and two roots which act as co-leaders of that cluster). Each root r_i is common in two adjacent clusters, (for example, see Figure 1 root r_2 is common in cluster S_1 and S_2). Each cluster S_i can be of different sizes and shapes with its corresponding complex Laplacian L_{S_i} and formation basis ξ_{S_i} . Every cluster satisfies algebraic and geometric conditions given in Lemma 2.3 and Lemma 2.4 which states that every cluster has stabilized complex Laplacian $D_{S_i} L_{S_i}$ associated with it. Now let us introduce a *meta-cluster* as following

Definition 3.1. A set of nodes, $V \in \mathcal{V}$, in cascade formulation is said to be meta-cluster, if one has

$$V = \{r_i | r_i, r_j \in \mathcal{V} \text{ and } (i, j) \in \mathcal{E}, \forall i \neq j; i, j \in \{1, 2, \dots, p\}\},$$

where r_i is the root of clusters and all nodes in V satisfy bidirectional 2-rooted graph topology.

The roots of meta-cluster act as main co-leaders. The orientation and scaling of the overall formation is decided by the formation basis of meta-cluster ξ_M . Moreover, the ξ_M are the location of the roots of clusters in complex coordinates. The stabilized complex Laplacian for meta-cluster is denoted by $D_M L_M$.

Proposition 3.2. Consider sufficiently large multi-agent network of n -agents having closed loop dynamics $\dot{z} = -DLz$, interconnected in cascade formulation. Then the eigenvalues of each cluster S_i and meta-cluster M are independent of each other if each cluster and meta-cluster satisfies 2-rooted graph topology and clusters are connected only through roots r_i .

Proof. Consider stabilized complex Laplacian is designed for each clusters and meta-cluster independently as DL_{S_i} and DL_M , respectively. This means that the clusters and meta-cluster satisfies conditions in Lemma 2.3 and Lemma 2.4. By using similarity transform of matrix to transform stabilized complex Laplacian into diagonal matrix with diagonal entries as corresponding eigenvalues [GVL12], [HJ12].

Let $P_{S_i} \in \mathbb{C}^{q_i \times q_i}$ and $P_M \in \mathbb{C}^{p \times p}$ be the matrices of right eigenvectors of cluster S_i , $i = 1, 2, \dots, p$ and meta-cluster M , respectively. The corresponding diagonal matrices can be represented as

$$(3.1) \quad \Lambda_{S_i} = P_{S_i}^{-1}(DL_{S_i})P_{S_i} = \text{diag}(0, \lambda_2^{S_i}, \lambda_3^{S_i}, \dots, \lambda_{q_i-1}^{S_i}, 0),$$

$$(3.2) \quad \Lambda_M = P_M^{-1}(DL_M)P_M = \text{diag}(0, \lambda_2^M, \lambda_3^M, \dots, \lambda_{p-1}^M, 0).$$

The 2-rooted graph topology of clusters and meta-cluster ensures that there exist two eigenvalues at origin corresponding to roots of the graph. The first and last row of Λ_{S_i} and Λ_M in (3.1) and (3.2) represents roots of clusters and meta-cluster, respectively. The proposed formulation considers that the adjacent clusters are connected only through its roots. This means that eigenvalues of cluster S_i will not affect the eigenvalues of clusters adjacent to it.

The 2-rooted structure of meta-cluster and Definition 3.1 insures that meta-cluster has $p - 2$ non-zero eigenvalues corresponding to roots of clusters r_i and remaining two zero eigenvalues will act as main co-leaders of overall network, thus it does not affect the other eigenvalues of clusters. \square

Remark 3.3. Proposition 3.2 implies that each cluster and meta-cluster can be treated as decoupled systems. Therefore, it is possible to design stabilized complex Laplacian DL for individual cluster and meta-cluster.

Theorem 3.4. Consider sufficiently large multi-agent network of n -agents interconnected in cascade formulation. The overall formation having closed loop dynamics $\dot{z} = -DLz$, results in a globally stable formation if individual cluster and meta-cluster are stabilized using complex Laplacian based control law.

Proof. The proof of this theorem is a direct consequence of Proposition 3.2. Assume that the individual stabilizing diagonal matrix D_{S_i} and D_M for every cluster and meta-cluster are designed to place eigenvalues of $D_{S_i}L_{S_i}$ and $D_M L_M$ to right hand side, respectively. As discussed in Subsection 3.2, each cluster and meta-cluster satisfies algebraic and geometric conditions.

Consider a cluster S_i with its corresponding formation basis ξ_{S_i} , the algebraic condition is

$$(3.3) \quad (D_{S_i}L_{S_i})\xi_{S_i} = 0, \quad \text{for } i = 1, 2, \dots, p$$

and the algebraic condition for meta-cluster M and formation basis ξ_M is

$$(3.4) \quad (D_M L_M)\xi_M = 0.$$

At roots of cluster there is an interaction between its adjacent clusters and meta-cluster, which gives

$$(3.5) \quad \sum_{i \in \Omega_{r_j}} (DL_{S_i}^{r_j})\xi_{S_i} + (DL_M^{r_j})\xi_M = 0, \quad \text{for } j = 1, 2, \dots, p,$$

where Ω_{r_j} is the set of adjacent clusters of root r_j , $DL_{S_i}^{r_j}$ and $DL_M^{r_j}$ represent row corresponding to r_j^{th} root of cluster S_i and meta-cluster M respectively. By using (3.3), (3.4) and (3.5), one has the algebraic condition of overall formulation for overall formation basis ξ as

$$(3.6) \quad (DL)\xi = 0.$$

In cluster-interaction (see Figure 1), there are p zero eigenvalues corresponding to its roots r_1, r_2, \dots, r_p . This gives rank of cluster interaction as $n - p$ and the meta-cluster has $n - 2$ non-zero eigenvalues at roots because of 2-rooted topology. Thus the overall rank of proposed formulation is $n - 2$. This satisfies the algebraic condition given in Lemma 2.3 to achieve globally stable formation. \square

Let the algebraic connectivity and the largest eigenvalue of p clusters be denoted by $\lambda_a(D_{S_1}L_{S_1}), \lambda_a(D_{S_2}L_{S_2}), \dots, \lambda_a(D_{S_p}L_{S_p})$ and $\lambda_{max}(D_{S_1}L_{S_1}), \lambda_{max}(D_{S_2}L_{S_2}), \dots, \lambda_{max}(D_{S_p}L_{S_p})$, respectively. For meta-cluster, it is represented as $\lambda_a(D_M L_M)$ and $\lambda_{max}(D_M L_M)$, respectively. As mention in Proposition 3.2, the eigenvalues of

each clusters are independent of others. Thus, the algebraic connectivity and the largest eigenvalue of overall formulation are given as

$$\begin{aligned}\lambda_a &= \min\{\lambda_a(D_{S_1}L_{S_1}), \lambda_a(D_{S_2}L_{S_2}), \dots, \lambda_a(D_{S_p}L_{S_p}), \lambda_a(D_M L_M)\}, \\ \lambda_{max} &= \max\{\lambda_{max}(D_{S_1}L_{S_1}), \lambda_{max}(D_{S_2}L_{S_2}), \dots, \lambda_{max}(D_{S_p}L_{S_p}), \lambda_{max}(D_M L_M)\}.\end{aligned}$$

Remark 3.5. *One can easily increase the hierarchy of formulation by cascading several meta-clusters whose formation basis is described by meta-meta-cluster without affecting the stability and performance of overall formation.*

Proposition 3.6. *The stabilized formation of sufficiently large multi-agent network using proposed formulation do not losses its overall stability even if it is subjected to uncertainties like actuation or communication link failure.*

Proof. The proposition is a direct consequence of Proposition 3.2, Remark 3.3, and Theorem 3.4. We have seen that the stability achieved in proposed formulation is clusterwise independent. Thus, even if one of the agent from a particular cluster losses its communication or actuation, it cannot affect the stability (i.e. eigenvalues) of adjacent clusters or meta-cluster. Thus the major part of large network remains undisturbed and stable. \square

3.3. Designing stabilization matrices. In proposed formulation, the stabilizing diagonal matrices D_{S_i} and D_M are used to stabilize and to improve its overall performance (like convergence time, control efforts, etc.) by controlling algebraic connectivity and fast transients. To achieve that, it is necessary to frame it into some optimization framework with constraints on eigenvalues. The conventional graph theory uses real valued Laplacian with positive, real and complex conjugate eigenvalues, however in the complex Laplacian approach, the parameter matrix itself is complex in nature and eigenvalues are randomly scattered in whole complex plane (not even complex conjugate in nature). To the best of our knowledge, the existing conventional optimization frameworks are not suitable to handle such complex parameter systems. Due to this reason, we opt for evolutionary algorithm based optimization technique can be implemented to design these parameters. In the proposed formulation, the matrices D_{S_i} and D_M are designed using genetic algorithm [SP94], [MTK96] with an objective of restricting the eigenvalue spectrum bandwidth of the complex Laplacian to a desired range.

The trade-off between convergence time and control efforts to avoid instability can be formulated in the form of objective function described as

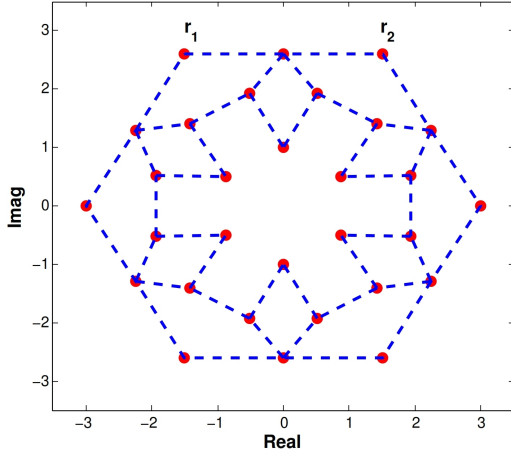
$$\min \begin{cases} \tau = f(\text{eig}(DL), \bar{\lambda}_{min}), \\ \sigma = g(\text{eig}(DL), \bar{\lambda}_{max}), \end{cases}$$

where τ represents an objective function for rate of convergence, σ denotes the objective function for control input intensity, $\bar{\lambda}_{min}$ and $\bar{\lambda}_{max}$ are lower and upper bounds on spectrum of nonzero eigenvalues of complex Laplacian.

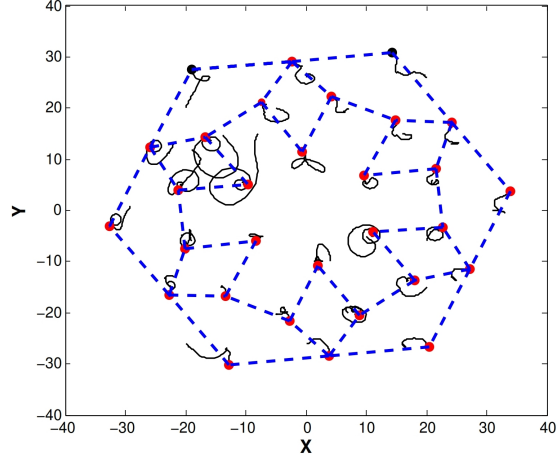
The objective function τ ensures that the all non-zero eigenvalues of clusters and meta-cluster are greater than required algebraic connectivity, λ_a , which controls the rate of convergence. The magnitude of control input u_i is restricted by bounding the eigenvalues below $\bar{\lambda}_{max}$ using objective function σ . This helps to avoid instability in case of saturation on control inputs. One can select value of $\bar{\lambda}_{max} > \lambda_a$ and close to the desired algebraic connectivity λ_a

4. SIMULATION AND RESULTS

In this section, we compare the simulation results to analyze performance and robustness of proposed formulation with conventional consensus algorithm using complex Laplacian.



(A) Target formation and interconnection between agents



(B) Closed-loop response using control law (2.3)

FIGURE 2. Formation control using conventional complex Laplacian

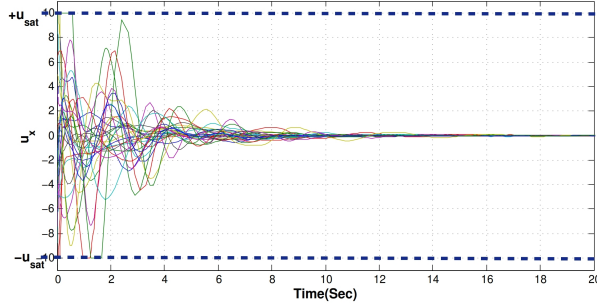
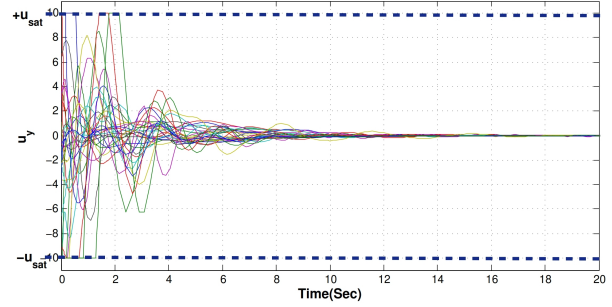
(A) Control input u_x without clustering(B) Control input u_y without clustering

FIGURE 3. Control inputs indicating convergence rate without clustering

4.1. Performance analysis. For the purpose of simulation, a system of 30 agents modelled by single integrator kinematics as given in (2.2) has been considered. The velocity input constraints are considered as $-10 \leq \text{Re}(u_i) \leq +10$ and $-10 \leq \text{Im}(u_i) \leq +10$. The system is simulated using MATLAB® Simulink. The target formation of agents represented by formation basis ξ in complex plane and interaction between agents is shown in Figure 2a. The black nodes indicate two roots of the 2-rooted graph \mathcal{G} . The simulation results using control law (2.3) is shown in Figure 2b. The stabilized complex Laplacian has its algebraic connectivity at $\lambda_3 = 0.0027 + \imath 0.1447$ and the largest eigenvalue at $\lambda_{max} = 16.44 - \imath 6.6786$. The observed convergence time and control signals are shown in Figure 3a and Figure 3b.

The proposed methodology is applied to a network discussed above. In accordance with the algorithm, the network of 30 agents is divided into six homogeneous clusters (S_1, S_2, \dots, S_6) as shown in Figure 4a. The black nodes represents the roots of clusters that comprise a meta-cluster. As all clusters are uniform, we can design same complex Laplacian L_S and Stabilizing matrix D_S for each cluster. The algebraic connectivity and largest eigenvalue of overall formation are at $1.5762 - \imath 3.4779$ and $23.352 - \imath 2.4619$ respectively. The response using proposed approach is given in Figure 4b. Figure 5a and 5b shows control signals u_x and u_y for simulation using cascade formulation. It is observed that the structured and distributed information flow due

to the proposed algorithm reduces the convergence time to around 5 sec while satisfying constraints on the control inputs.

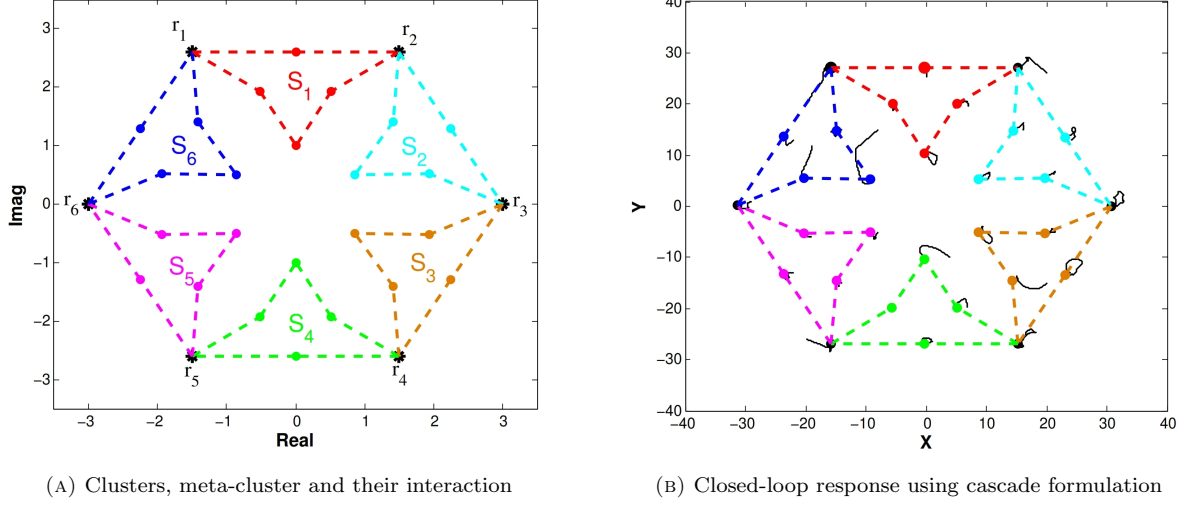


FIGURE 4. Formation control using cascade formulation

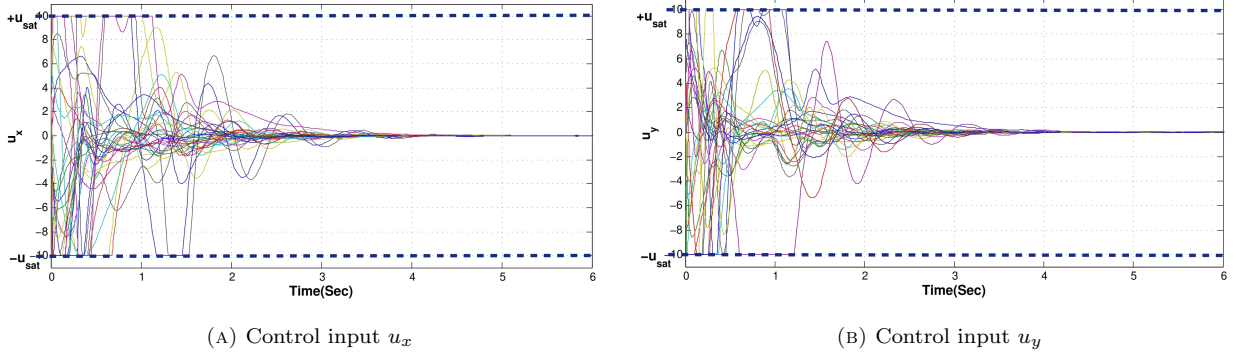


FIGURE 5. Control inputs indicating convergence rate using proposed formulation

4.2. Robustness to communication and actuation failure. We now illustrate the robustness property of proposed formulation in case of failure in any communication link or actuator of an agent. It is easily observed from stability conditions that the conventional complex Laplacian based control law causes instability due to communication interruption and actuation failure.

The proposed formulation incorporates the stability of individual cluster and meta-cluster. Due to this, the communication and actuation failure of any agent of a particular cluster will not affect the stability of other clusters. To illustrate the results for communication link failure, we have considered two cases. In first case, the failure of communication link of an agent within cluster is considered and the simulation result is shown in Figure 6a. The result shows that the failure will not affect the stability of other clusters and meta-cluster. In second case, failure of link in meta-cluster agent is simulated and it is observed that all clusters approach stable formation. The failure only affects the orientation and scaling of clusters adjacent to affected link (see Figure 6b).

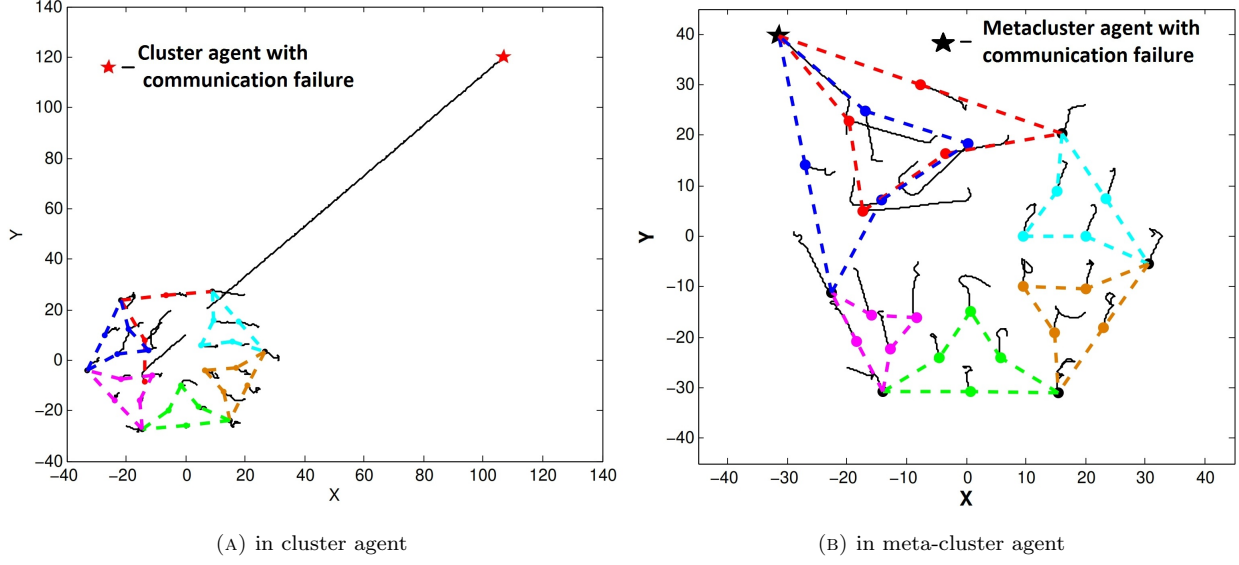


FIGURE 6. Response due to loss of communication

The simulation result for actuation failure of an agent in cluster and meta-cluster is shown in Figure 7a and 7b, respectively. It is observed that the proposed formulation forms stable formation around the agent with actuation failure,

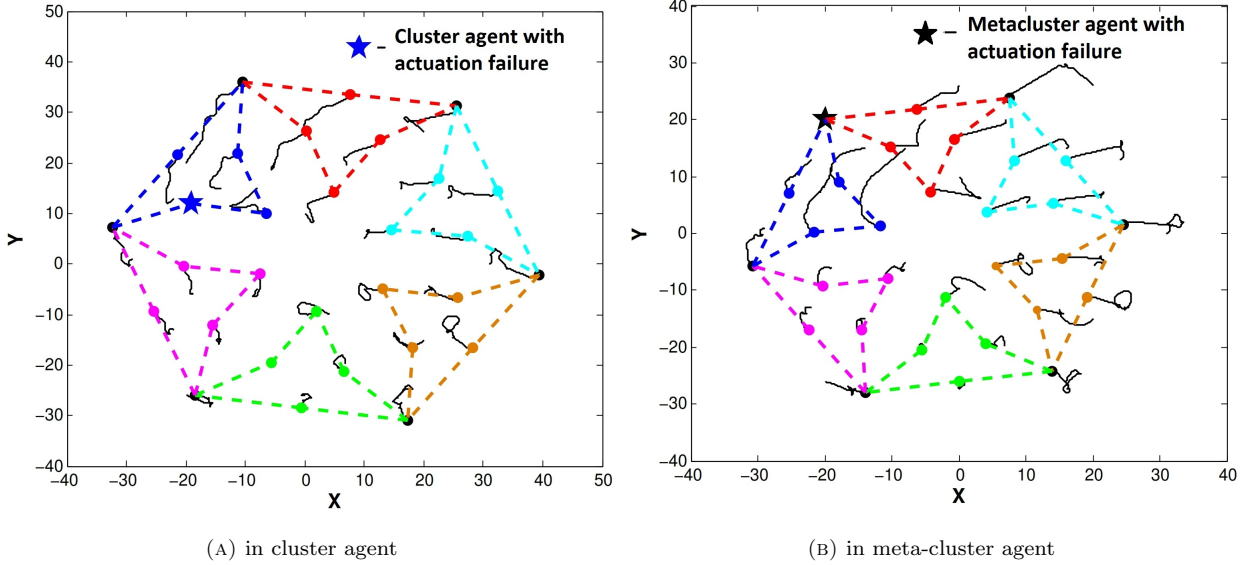


FIGURE 7. Response due to actuation failure

5. CONCLUSIONS AND OPEN PROBLEMS

A novel approach is formulated to solve formation control problem in large multi-agent systems while attaining robustness to communication link and actuation failure. The cascade formulation proposed in this paper

channelizes information flow throughout the network efficiently. The formulation divides the complex network into small clusters to incorporate decentralized information exchange between meta-cluster and agents of individual clusters. The 2-rooted bidirectional graph topology is adopted to form clusters and meta-cluster which allows them to be a decoupled dynamical systems. This offers flexibility in designing individual control laws, that satisfies the bounds on control inputs and achieves stable formation in desired convergence time. Moreover, it is illustrated that the proposed formulation is relatively robust even if the information flow in network is subjected to uncertainties like communication link and actuation failure in agents. The cascade formulation also provides for organization of distributed clusters at different hierarchies in complex systems, which is helpful in many applications like synchronization and collective task handling in multi-robot systems.

As an extension to the proposed methodology, our future research is focused on some challenging issues like collision avoidance, self organizing cascade formulation, etc. Moreover, the control strategy can be cast into well structured optimization framework to design a stabilizing diagonal matrix for optimization of control efforts and convergence time.

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